



APRIL/MAY 2025

GMA32/DMA32 — TOPOLOGY

17. Let $\{X_\alpha\}$ be an indexed family of spaces. Let $A_\alpha \subset X_\alpha$ for each α . If $\prod x_\alpha$ is given either the product or the box topology then $\overline{\prod A_\alpha} = \prod \overline{A_\alpha}$.
18. If L is a linear continuum in the order topology, then prove that L is connected.
19. Prove that every closed interval in R is compact.
20. State and prove Urysohn Lemma.

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define product topology.
2. Define closed set of a topological space with example.
3. Define metrigable space.
4. State sequence lemma.
5. Define locally connected.
6. Define path components of x .
7. What is finite intersection property?
8. State uniform continuity theorem.
9. Define Lindelof space.
10. What do you say a space is a first countable space?

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Prove that the topologies of $\mathbb{R}_\ell$ and $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$, but not comparable with one another.

Or

- (b) If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y then the collection $D = \{B \times C / B \in \mathcal{B} \text{ and } C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$ — Prove.
12. (a) Prove that the topologies of $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.

Or

- (b) Prove that the composition of two continuous functions is continuous.
13. (a) Prove that a finite Cartesian product of connected spaces is connected.

Or

- (b) Prove that a space X is locally connected if and only if for every open set U of X , each component of U is open in X .

14. (a) Prove that the product of finitely many compact spaces is compact.

Or

- (b) State and prove Lebesgue number Lemma.

15. (a) Prove that a subspace of a regular space is regular and product of regular spaces is regular.

Or

- (b) Prove that every well ordered set X is normal in the order topology.



SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let X be a topological space. Then the following conditions hold

- (a) $\emptyset$ and x are closed
 (b) Arbitrary intersection of closed sets are closed.
 (c) Finite unions of closed sets are closed.